

Compensation for extreme outages caused by polarization mode dispersion and amplifier noise

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Abstract: Fluctuations of Bit-Error-Rate (BER) stimulated by birefringent disorder in an optical fiber system are found to be strong. The effect may not be analyzed in terms of the average BER but rather requires analyzing the Probability Distribution Function (PDF) of BER. We report the emergence of the extremely extended algebraic-like tail of the PDF, corresponding to anomalously large values of BER. We analyze the dependence of the PDF tail, and thus the outage probability, on the first-order PMD compensation scheme. Effectiveness of compensation is illustrated quantitatively using a simple, however, practical example.

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References and links

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1. Introduction

Polarization Mode Dispersion (PMD) is recognized to be a substantial impairment for optical fiber systems with the 40-Gbs/s and higher transmission rates. One may not have a complete control over PMD since the fiber system birefringence is changing substantially with time under the influence of environmental condition (e.g., stresses and temperature) fluctuations, see e.g., [1, 2]. Thus, dynamical PMD compensation became a major issue in modern optical fiber telecommunication technology [3, 4]. Development of experimental techniques capable of the first- [5, 6, 7] and higher-orders [7, 8] PMD compensation have raised a question of how to evaluate the compensation success (or failure). Traditionally, the statistics of the PMD vectors of first [9, 10, 11] and higher orders [12, 13, 14] is considered as a measure for any particular compensation method performance. However, these objects are only indirectly related to what actually represents the fiber system reliability. In this letter we show that the PMD effects should be considered only together with the impairments due to amplifier noise, since fluctuations of BER caused by variations of the birefringent disorder, are substantial. We demonstrate that the probability of extreme outages is much larger than one could expect from naive estimates singling out effects of either of the two impairments. This phenomenon is a consequence of a complex interplay between the impairments of different natures: Birefringent disorder is frozen, i.e., it does not vary on all propagation-related time scales, while the amplifier noise is short-correlated. The effect may not be explained in terms of just an average value of BER, or statistics of any PMD vectors of different orders, but rather should be naturally described in terms of the PDF of BER, and specifically its tail. A consistent theoretical approach to calculating the tail will be briefly sketched, with a prime attention given to the analysis of the effects of the first- and higher-order compensation on the extreme outages measured in terms of the PDF of BER.

2. Bit-Error-Rate

We consider the so-called return-to-zero modulation format, when pulses (information carriers) are well separated in time t . The quantity measured at the output of the optical fiber line is the pulse intensity:

$$I = \int dt G(t) |\mathcal{K}\Psi(Z, t)|^2, \quad (1)$$

where $G(t)$ is the convolution of the electrical (current) filter function with the sampling window function. The two-component complex field $\Psi(Z, t)$ describes the output signal envelope. The two components correspond to two polarizations of the optical fiber mode. The linear operator \mathcal{K} in Eq. (1) stands for a variety of engineering “tricks” applied to the output signal. They consist of the optical filter \mathcal{K}_f , and the compensation \mathcal{K}_c parts, respectively, assuming the compensation is applied first followed by filtering, i.e. $\mathcal{K} = \mathcal{K}_f \times \mathcal{K}_c$. Ideally, I accepts two different values depending on whether the information slot is vacant or filled. However, the impairments enforce deviations of I from their ideal values. Therefore, one has to introduce a threshold (decision level) I_d and declare that the signal encodes “1” if $I > I_d$ and is related to “0” otherwise. Sometimes the information is lost, i.e. an initial “1” is detected as a “0” at the output or vice versa. BER is the probability of such an “error” event (with the statistics collected over many pulses coming through a fiber with a given realization of birefringent disorder). For successful system performance the BER must be extremely small, i.e. both impairments typically cause only small distortions to a pulse. It is straightforward to verify that anomalously high values of BER originate solely from the “1 \rightarrow 0” events. We denote the probability of such events by B . Note, that errors associated with pulse migration/jitter from neighboring information slots are not considered here. These migration effects that should be definitely taken into account in the Non-Return-To-Zero (NRZ) modulation format case are strongly suppressed for the RZ modulation format since the ratio of the pulse width to the slot width is small. Non-zero

B is actually caused by the noise, the value, however, depending on a particular realization of the birefringent disorder.

3. Noise averaging

We consider the linear propagation regime, when the output signal Ψ can be decomposed into two contributions: φ , related to a noiseless initial pulse evolution and the noise-induced ϕ part of the signal. ϕ appears to be a zero-mean Gaussian variable (insensitive to a particular realization of birefringence and chromatic dispersion in the fiber) and is fully characterized by the pair correlation function

$$\langle \phi_\alpha(Z, t_1) \phi_\beta^*(Z, t_2) \rangle = D_\xi Z \delta_{\alpha\beta} \delta(t_1 - t_2). \quad (2)$$

Here, Z is the total fiber line length, and the product $D_\xi Z$ is the amplified spontaneous emission (ASE) spectral density accumulated along the system. The coefficient D_ξ is introduced into Eq. (2) to reveal the linear growth of the ASE factor with Z [15].

4. Disorder averaging

The noise-independent part of the signal is

$$\varphi = \exp \left(i \int_0^Z dz d(z) \partial_t^2 \right) \hat{U} \Psi_0(t), \quad \hat{U} = T \exp \left[\int_0^Z dz \hat{m}(z) \partial_t \right], \quad (3)$$

where $\Psi_0(t)$, z , and $d(z)$ are the input signal profile, coordinate along the fiber, and the local chromatic dispersion, respectively. The ordered exponent \hat{U} depends on the 2×2 matrix $\hat{m}(z)$ that characterizes the birefringent disorder. The matrix can be represented as $\hat{m} = h_j \hat{\sigma}_j$, $h_j(z)$ being a real three-component field and $\hat{\sigma}_j$ the Pauli matrices. Averaging over many states of the birefringent disorder any given fiber is going through (birefringence varies on a time scale much longer than any time scale related to the pulse propagation through the fiber or, equivalently instant states of birefringence in different fibers, one finds that $h_j(z)$ is a zero-mean Gaussian field described by the following pair correlation function

$$\langle h_i(z_1) h_j(z_2) \rangle = D_m \delta_{ij} \delta(z_1 - z_2). \quad (4)$$

If birefringent disorder is weak the integral $\mathbf{H} = \int_0^Z dz \mathbf{h}(z)$ coincides with the PMD vector. Thus, $D_m = k^2/12$, where k is the so-called PMD coefficient.

H-dependence of BER. For successful fiber system performance the BER should be extremely small, i.e. typically both impairments can cause only small distortions of a pulse. Stated differently, the optical signal-to-noise ratio (OSNR) and the ratio of the squared pulse width to the mean square value of the PMD vector are both large. OSNR can be estimated as $I_0/(D_\xi Z)$ where $I_0 = \int dt |\mathcal{K}_f \Psi_0(t)|^2$, and the integration goes over a single slot populated by an ideal (initial) pulse, encoding “1”. Since the value of OSNR is large averaging over the noise can be performed using the saddle-point method. This leads to a conclusion that $D_\xi Z \ln B$ depends on the birefringence, shape of the initial signal and the details of the compensation and measurement procedures, being, however, independent of the noise. Typically, B fluctuates around B_0 , the zero-disorder ($h_j = 0$) value of B . For any finite value of \mathbf{h} one gets, $\ln(B/B_0) = \Gamma I_0/(D_\xi Z)$, where the dimensionless factor Γ depends on \mathbf{h} . Since the noise is weak, even small disorder can generate strong increase in B . This implies that a perturbative calculation of Γ based on expanding the ordered exponent \hat{U} in Eq. (3) in powers of \hat{m} , describes the most essential part of the PDF of B . Thus, in the situation when no compensation is applied one derives $\Gamma = \mu_1 H_3/b$, whereas in the simplest case of the “setting the clock” compensation, accounting for the average (typical) temporal shift (this corresponds to the change of t in Eq.(1) to $t - t_{cl}$, where

the t_{cl} is the z -independent shift dependent on the birefringence profile $\mathbf{h}(z)$, one arrives at $\Gamma = \mu_2(H_1^2 + H_2^2)/b^2$, b and $\mu_{1,2}$ being the pulse width and some dimensionless coefficients, respectively.

5. Long tail

The PDF $\mathcal{S}(B)$ of B (that appears as a result of averaging over many realizations of the birefringent disorder) can be found by recalculating the statistics of H_j using Eq. (4) followed by substituting the result into the corresponding expression that relates B to H_j . Our prime interest is to describe the PDF tail that corresponds to the values of H_j substantially exceeding their typical value $\sqrt{D_m Z}$ remaining, however, much smaller than the signal duration b . In this range one gets the following estimates for the differential probability $\mathcal{S}(B)dB$:

$$a) \exp \left[-\frac{D_\xi^2 Z b^2}{2D_m \mu_1^2 I_0^2} \ln^2 \left(\frac{B}{B_0} \right) \right] \frac{dB}{B}, \quad b) \frac{B_0^\alpha dB}{B^{1+\alpha}}, \quad (5)$$

where (a) corresponds to the no-compensation situation, (b) stands for the optimal “setting the clock” case, and $\alpha \equiv D_\xi b^2 / (2\mu_2 D_m I_0)$. Note, that the result in the case (b) shows a steeper decay compared to the case (a), which is a natural consequence of the “setting the clock” compensation.

6. PMD compensation

Effects of PMD can be reduced by using a device usually called a PMD compensator (PMDC). Any PMDC consists of two parts: a compensating (optical) part responsible for the compensation itself, and a measuring part that extracts (measures) relevant information on the transmission fiber birefringence. The compensator of N -th order consists of N pieces of polarization-maintaining fiber (i.e., described by uniform, position independent, birefringence vector) usually surrounded by two polarization controllers, that allow rotation of the polarization state [6]. This implies that the optical part of a PMDC can be characterized by its transfer function. Additionally, one would naturally distinguish between (i) describing a compensator in terms of available transfer functions, and (ii) compensating strategy, i.e. a prescription of how to fix the compensating degrees of freedom based on the measured data. The first order PMD compensator corresponds to $\mathcal{K}_c = \mathcal{K}_1$,

$$\mathcal{K}_1(\mathbf{M}) = \exp(-M_j \hat{\sigma}_j \partial_t), \quad (6)$$

with $j = 1, 2, 3$. Such a form of the compensating operator \mathcal{K}_c offers richer adjustment options compared to the “setting the clock” compensation as it actually contains three compensating degrees of freedom, i.e. the three components of the compensating vector \mathbf{M} , instead of one. Note also, that the transfer matrix \hat{U} of the transmission fiber is defined as an ordered exponential (3), whereas the compensating operator \mathcal{K}_1 is defined in terms of the usual exponential (6), simply because birefringence is uniform along the compensating part but varies significantly along the transmission fiber. The standard PMD compensation strategy, discussed in the literature, boils down to compensating for as many terms as possible in the ordered exponential \hat{U} expansion in the series in \mathbf{h} [12, 13, 14]. Therefore, in the first-order compensator case (6) one chooses, $\mathbf{M} = \int_0^Z dz \mathbf{h}(z)$, to ensure the expansion of $\mathcal{K}_1 \hat{U} - \hat{1}$ in a series in \mathbf{h} starts with the $O(h^2)$ terms only. Expanding $\mathcal{K}_1 \hat{U}$ in \mathbf{h} followed by substituting the result into Eq. (1) and evaluating B leads to

$$\Gamma = (\mu_2'/b^2) \int_0^Z dz' \int_0^{z'} dz [h_1(z')h_2(z) - h_2(z')h_1(z)], \quad (7)$$

b being the pulse width, and only the leading $O(h^2)$ term is retained in Eq. (7). The dimensionless coefficient μ'_2 is related to the output signal chirp, produced by the initial signal chirp and/or the nonzero integral chromatic dispersion η . Recalculating the statistics of Γ using Eqs. (4,7) one obtains the following tail $\mathcal{S}(B)$ for the PDF of B

$$\mathcal{S}(B)dB \sim \frac{B_0^\gamma dB}{B^{1+\gamma}}, \quad \gamma \equiv \frac{\pi D_\xi b^2}{2|\mu'_2|D_m I_0}, \quad (8)$$

and Eq. (8) holds when $\ln(B/B_0) \gg |\mu'_2|D_m I_0/[D_\xi b^2]$.

7. Non-chirped signal

If the output signal is not chirped then $\mu'_2 = 0$ and the first non-vanishing term in the expansion of Γ in h_j is of the third order. Expanding $\mathcal{K}_1 \hat{U}$ up to the leading term yields

$$\Gamma = \frac{\mu_3}{b^3} \int_0^Z dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \left\{ 2h_3(z_1)\mathcal{H}(z_2, z_3) - h_3(z_2)\mathcal{H}(z_1, z_3) - h_3(z_3)\mathcal{H}(z_1, z_2) \right\}, \quad (9)$$

with $\mathcal{H}(z_1, z_2) = h_1(z_1)h_1(z_2) + h_2(z_1)h_2(z_2)$. Substituting Eq. (9) into the expression for B in terms of Γ and making use of Eq. (4) leads to a representation of the PDF of B as a path-integral over \mathbf{h} . Integrating over h_3 explicitly and approximating the resulting integral over $h_{1,2}$ by its saddle-point value, one finds the PDF tail

$$\ln \mathcal{S} \approx -4.2 \frac{(D_\xi Z/I_0)^{2/3} b^2}{\mu_3^{2/3} D_m Z} \left(\ln \frac{B}{B_0} \right)^{2/3}. \quad (10)$$

Eq. (10) is valid at $D_\xi Z \ln(B/B_0) \gg \mu_3 (D_m Z)^{3/2} I_0 / b^3$.

8. Simple model

The dimensionless coefficients μ'_2 , μ_3 can be computed in the framework of a simple model, with the decision level threshold I_d being twice smaller than the ideal intensity, the Lorentzian profile of the optical filter $\mathcal{K}_f \Psi = \int_0^t dt' \exp[-t/\tau] \Psi(t-t')/\tau$, and the step function form for $G(t) \propto \theta(T - |t|)$. (Note, that the optimal choice of I_d and the shapes of the optical and electrical filters in practical implementations constitutes a bit more sophisticated procedure.) We also consider a Gaussian weakly-chirped initial signal $\Psi_0 \propto \exp(-t^2/[2b^2])(1 + i\beta_{in}t^2/b^2)$, $\beta_{in} \ll 1$. The output signal chirp becomes $\beta = \beta_{in} + \eta$, η being the integral dimensionless chromatic dispersion. Then, μ'_2 is proportional to β , and the slope μ'_2/β , found from the saddle-point equations numerically, along with corresponding values of $\Gamma_0 \equiv -D_\xi Z \ln B_0 / I_0$ and $\mu_{1,2,3}$ are shown in Fig. 1 for a reasonable range of the width of the temporal slot T and optical filter width τ , both measured in the units of the pulse width b .

9. Example

Summarizing, our major result is quantitative description of the suppression of the extremely long tail in the PDF of BER: Eqs.(5,10,8) constitute an explicit set of expressions that allow to compute the outage probability, defined as $\mathcal{O} = \int_{B_*}^1 dB \mathcal{S}(B)$, with B_* being some fixed value much larger than B_0 . We find it useful to conclude with presenting a numerical example that corresponds to a case relevant for the optical fiber communications. We choose the pulse width $b = 25ps$, the electric filter width $T = 1.25 \cdot b$ and the optical filter width $\tau = 0.5 \cdot b$. According to Fig. 1 this corresponds to the following set of dimensionless coefficients: $\Gamma_0 \approx 0.06$, $\mu_1 \approx -0.06$, $\mu_2 \approx 0.12$, $\mu'_2/\beta \approx 0.1$ and $\mu_3 \approx 0.25$. The value of Γ_0 corresponds to the signal-to-noise ratio $I_0/[D_\xi Z] \approx 460$, and typical BER $B_0 \approx 10^{-12}$. We also assume that the PMD coefficient,

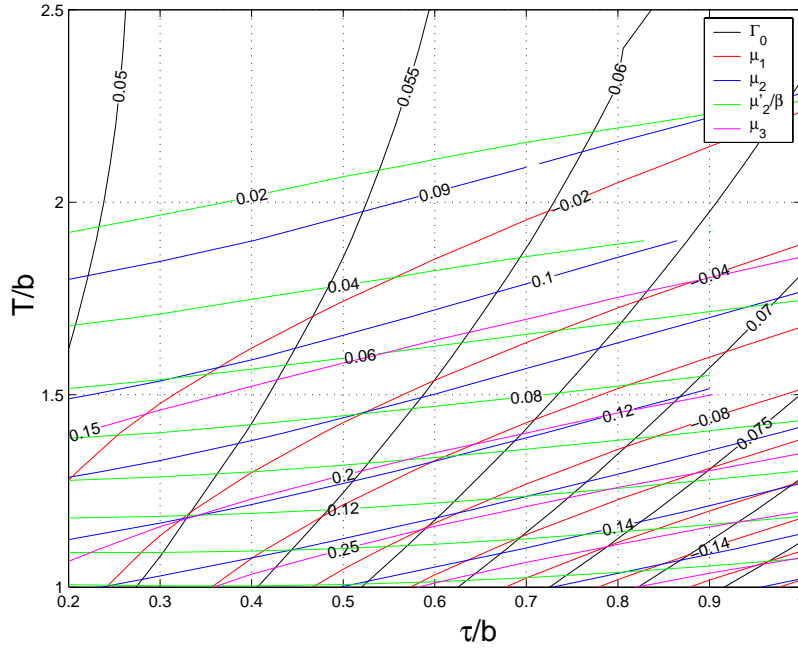


Fig. 1. Dependence of the dimensionless coefficients $\Gamma_0 = -D_{\xi} z \ln B_0 / I_0$, μ_1 , μ_2 , μ_2' / β and μ_3 , entering Eqs.(5,8,10) on the slot size T and the optical filter width τ , both measured in the units of the pulse width b . The coefficients are calculated numerically using the simple model explained in the text.

$k = \sqrt{12D_m}$, is $0.2 \text{ ps}/\sqrt{\text{km}}$, and the system length is $Z = 3,000 \text{ km}$, i.e. $D_m Z / b^2 \approx 0.016$. The outage probability corresponding to $B_* = 10^{-10}$, i.e. the probability for B to be at least 2 orders of magnitude larger than B_0 , is $\mathcal{O} \approx 0.41$ if no compensation is applied [see Eq. (5a)], while one derives $\mathcal{O} \approx 0.07$, $\mathcal{O} \approx 5 \cdot 10^{-5}$ and $\mathcal{O} \approx 5 \cdot 10^{-14}$ for Eq. (5b), Eq. (8) and Eq. (10) that describe the cases of the “setting the clock”, first- and second-order compensation, respectively. Thus, if a typical scale of the birefringence change is $0.1s$ one should expect an outage in $0.25s$, $1.5s$, $2,000s \approx 1/2 \text{ hour}$ and $2 \cdot 10^{12}s \approx 60,000 \text{ years}$ if no compensation, setting the clock compensation and first-order compensation with chirped and non-chirped output signal, respectively, is applied.

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